TREATMENT OF INSTABILITIES OF ONE-WAY EQUATION ABSORBING BOUNDARY CONDITIONS USING DIGITAL FILTERS

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ABSTRACT

This paper presents a general solution to the difficult problem of treating instabilities of one-way equation (local) absorbing boundary conditions in time domain electromagnetic simulations. Digital filters have been inserted between the mesh and the absorbing boundaries to filter out the high frequency numerical noise generated by the absorbing boundary interpolation equations. The problem arising out of the finite delay of the digital filters has been solved my making the filter delay equal to the time delay in the longer link lines of the graded mesh. By adopting this technique, instabilities have been eliminated.

INTRODUCTION

One-way equation absorbing boundary conditions are very commonly used in the time-domain analysis of electromagnetic structures using transmission line matrix (TLM) and finite difference time domain (FDTD) methods [1-3]. They are inexpensive to implement because they make only use of the fields at the neighboring space and time nodes. However, one problem associated with the one-way equation or local absorbing boundaries is instability. These instabilities are usually small in the early time; they grow as computations proceed, and at later times, they form the dominant part of the solution.

Instabilities have been controlled to some extent using higher precision in computations, using bandlimited excitation, adding damping factors in the boundary operators, and by choosing proper finite differences for the boundary operators [1, 2, 4, 5]. But application of these is very tricky and problem dependent. There is no single solution scheme which gives stable solutions at all times and for all structures. In this paper, we propose a new general scheme to control the instabilities. We use digital filters to filter out the unwanted high frequency noise generated by the absorbing boundaries. The main difficulty in the implementation of this scheme is the finite amount of delay introduced by the digital filters. We have used a variable mesh scheme known as memory technique [6] to circumvent this problem very effectively and naturally. Earlier digital filters have been used to design the absorbing boundaries [7].

THEORY

A two-dimensional graded TLM mesh with longer link lines in the vicinity of the absorbing boundaries is shown in Fig.1. N is the grading ratio. The time step Δt is taken as the time required to travel the length of the shortest link line (Δl_1). The phase velocities on all the link lines in the mesh are kept the same. Hence an impulse takes $N\Delta t$ to travel on the longer lines. The impulses travelling on the longer branches have to be kept in store for N time steps before being injected into the next node. In this paper, we exploit this feature of the variable mesh to offset for the delay introduced by the digital filters.

For demonstration purposes, we have chosen a second-order one-way equation absorbing boundary condition based on Higdon's formulation[1-2]. A voltage impulse reflected from the absorbing boundary can be computed from the knowledge of impulses in the cells in front of the boundary using the following equation:

$$\begin{aligned} V^n(j,k) &= (\alpha_1+\alpha_2)V^{n-1}(j,k)-\alpha_1\alpha_2V^{n-2}(j,k) \\ &+(\beta_1+\beta_2)V^n(j-1,k) + (\gamma_1+\gamma_2-\alpha_1\beta_2-\beta_1\alpha_2) \\ &\quad V^{n-1}(j-1,k)-(\alpha_1\gamma_2+\gamma_1\alpha_2)V^{n-2}(j-1,k) \\ &-\beta_1\beta_2V^n(j-2,k)-(\beta_1\gamma_2+\gamma_1\beta_2)V^{n-1}(j-2,k) \\ &-\gamma_1\gamma_2V^{n-2}(j-2,k) \end{aligned} \tag{1}$$

where $V^n(j,k)$ represents the voltage impulse reflected from the node ($j\Delta x$, $k\Delta y$) at time $n\Delta t$.

The interpolation coefficients are:

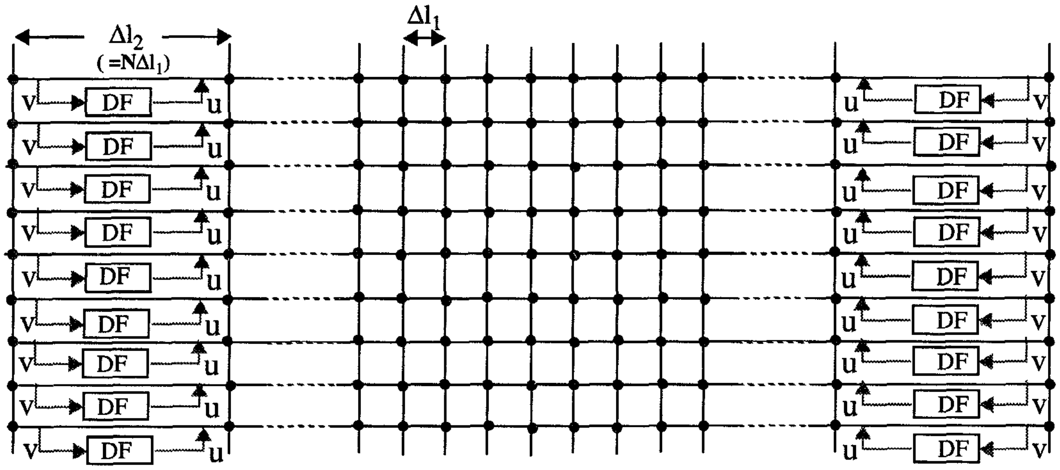

Fig.1: Two-dimensional graded TLM mesh and digital filters (DF) to filter out the high frequency noise generated by the absorbing boundary interpolation equations. v is computed using absorbing boundary interpolation equations. u is the output of the digital filters.

$$\alpha_i = \frac{(a - g_i(1-b))}{a - 1 - g_i(1-b)}$$

$$\beta_i = \frac{(a - 1 + g_i b)}{a - 1 - g_i(1-b)}$$

$$\gamma_i = \frac{(-a - bg_i)}{a - 1 - g_i(1-b)} \quad (2)$$

where coefficients a and b are weighted time and space averages of the space and time differences, respectively. θ_i is the incidence angle. The parameter g_i for the uniform mesh is

$$g_i = \frac{\cos\theta_i}{c}\frac{\Delta l}{\Delta t} \quad (3)$$

while for the variable mesh, it can be written as

$$g_i = \sqrt{2}\,\frac{\Delta l_2}{\Delta l_1}\cos\theta_i \quad (4)$$

Next, we pass the time samples $V^n(j,k)$ computed from the above equation through an IIR digital filter. The output $U^n(j,k)$ of the digital filter is obtained from the following equation:

$$U^n(j,k) = \sum_{m=0}^{L} b_m V^{n-m}(j,k) - \sum_{m=1}^{L} a_m U^{n-m}(j,k) \quad (5)$$

where a_m and b_m are the digital filter coefficients, and L is the order of the filter. The output $U^n(j,k)$ of the digital filter will be delayed by the finite delay of the filter. This delay depends upon the factors such as the bandwidth of the passband, attenuation in the stopband, etc. These factors should be chosen such that the phase response of the filter is linear in the frequency band of interest. If this delay is made equal to the grading ratio, N, the impulses $U^n(j,k)$ do not have to be kept in store for N time steps before being injected into the next node, and they can be directly fed to the next node.

NUMERICAL RESULTS

A section of WR 28 waveguide has been considered with absorbing boundaries at the input and output ends. Using magnetic wall symmetry, only one-half of this structure was discretized with a TLM mesh of size (100x73). The length of the shortest link line is 0.05 mm. The lengths of the link lines in the first 10 and the last 10 cells (in the main propagation direction) are five times longer, i.e. 0.25 mm. The time step Δt is 0.11785 ps. The filter was excited with a cosine modulated Gaussian pulse. The carrier frequency and the width of the Gaussian pulse are chosen such that only the frequencies that cover the passband of the TE_{10} mode of the WR28 wave guide are excited. The incidence angles used for the absorbing boundaries are 30^0 and 60^0.

A digital filter that suppresses all the high frequency noise and that has a delay of $5\Delta t$ (equivalent phase value at 30 GHz is 6.363 degrees) should be designed. The coefficients of such a digital filter are the following [8]:

$$b_0 = 0.0879,\ b_1 = -0.1146,\ b_2 = 0.08797$$
$$a_1 = -1.6334, a_2 = 0.6947 \quad (6)$$

The phase response of this digital filter is plotted in Fig. 2. We can see that the phase response is linear at least upto 100 GHz, and the delay of this filter is 5Δt, i.e., 0.5892 ps. The input and output time samples of the filter are plotted in Fig. 3.

Figs. 4a and 4b show the time domain responses of the waveguide computed without using the digital filters for different sets of coefficients *a* and *b*. Figs. 5a and 5b show the corresponding responses obtained with the digital filters present. We can clearly see that with the digital filters, the responses are always stable and there is no instability problem, while in the absence of digital filters, the onset of instability depends on the type of finite differences (values of *a* and *b*) used in the boundary operators.

CONCLUSIONS

Digital filters have been used to obtain very stable absorbing boundaries. The problem of accounting for the finite delay of the digital filters has been solved by using longer link lines in the vicinity of the absorbing boundaries. As any microwave component will have input and output uniform transmission lines, using a coarse mesh (longer lines) in these sections will reduce the total number of cells without compromising the accuracy. This technique leads to very stable and robust time domain simulation techniques. Although the idea has been tested in a two-dimensional TLM algorithm, it can also be applied to the FDTD method.

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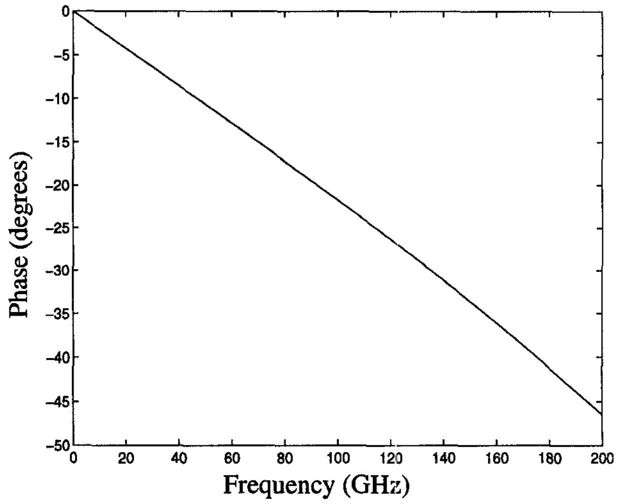

Fig.2: Phase response of the digital filter.

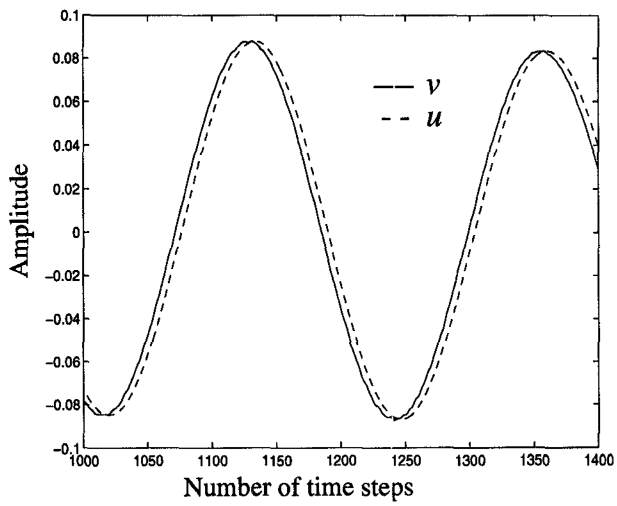

Fig.3: Plot of input (v) and output (u) of the digital filter

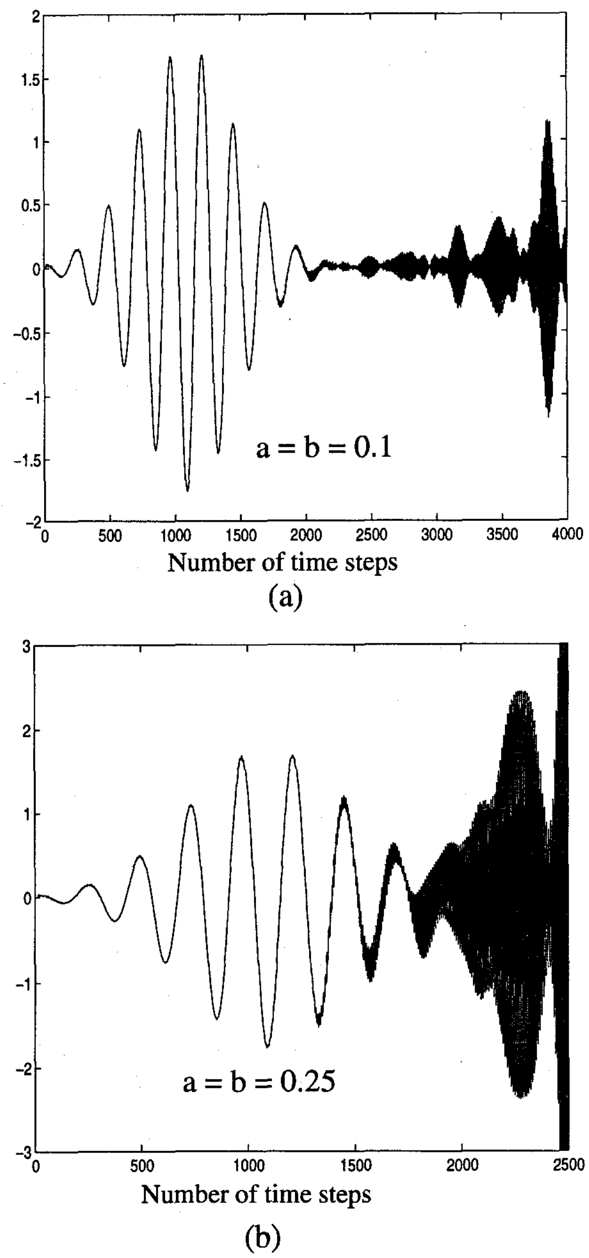

Fig.4: Time domain responses of the waveguide discretized with a variable mesh, and without digital filters.

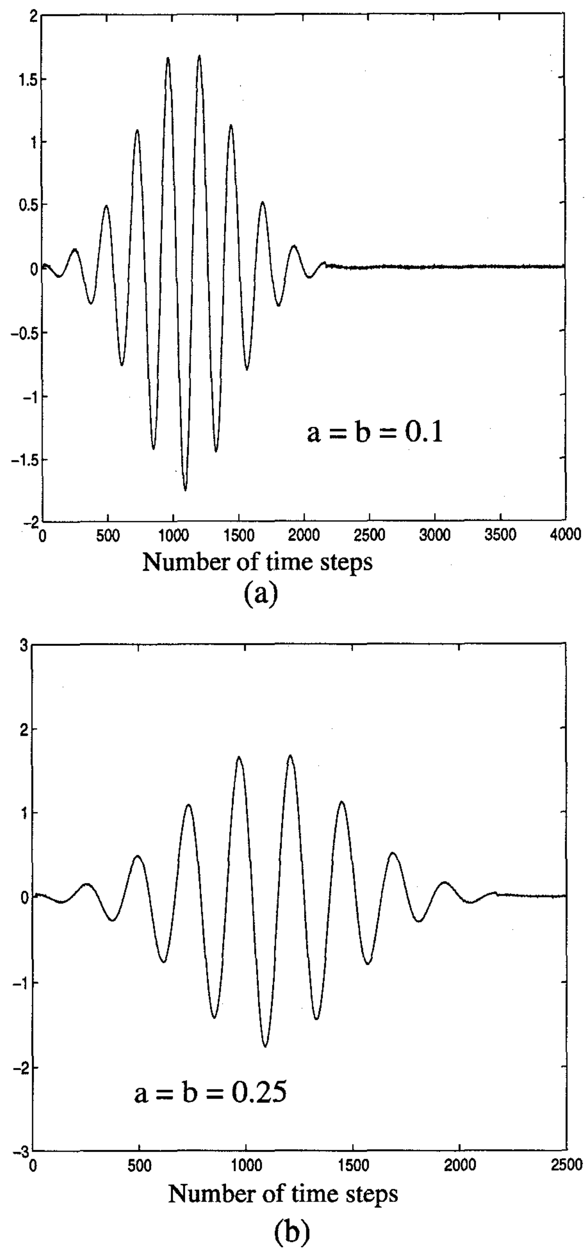

Fig.5: Time domain responses of the same waveguide with digital filters inserted between mesh and absorbing boundaries.

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